APPLICATION OF A LINEAR PROGRAMMING MODEL TO GRAIN MERCHANDISING

by

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TABLE OF CONTENTS

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LIST OF TABLES

LIST OF TABLES
(continued)

LIST OF ILLUSTRATIONS

INTRODUCTION

Decision-making, the process of choosing from among a number of alternatives, is a basic characteristic of management. Decisions are continually made concerning the means of achieving objectives of the firm in every industry. Management requires information in order to make sound decisions. In almost every situation there is a considerable amount of information available. The problem is to choose the relevant information and determine the appropriate use of this information on reaching the goals of the firm.

It is increasingly evident that a systematic approach to the formulation and solution of business problems is an explicit characteristic of modern management.[1] One of the most popular and useful techniques that management can turn to for assistance is the mathematical technique of linear programming. Linear programming is a scientific method for selecting an optimum solution to a problem by solving simultaneously a set of equations or inequalities under specific mathematical conditions.[2]

The purpose of this study is to investigate the application of linear programming as a tool to assist management decision

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[1] Thomas H. Naylor and Eugene T. Byrne, Linear Programming--Methods and Cases (Belmont, Calif.: Wadsworth Pub. Co., 1963), p. 4.

[2] Leonard W. Schruben, "Mathematical Models for Decision and Control in Flour Milling," Association of Operative Millers Technical Bulletin, (August, 1967), p. 2989.

making by grain merchandising firms from a profit maximizing point of view. The objective is to formulate a linear programming model which can be used to develop grain merchandising policy and assist management in making merchandising decisions. The mathematical theory of linear programming will not be discussed in this paper.

Considerable material has been published on the application of linear programming in the feed manufacturing industry where it has become a widely accepted technique used primarily in determining least cost feed formulas. Baynham[3] and Clithero[4] have suggested the application of linear programming to grain merchandising but again with the idea of filling orders at least cost. Unger[5] presented a linear programming grain merchandising problem in which he considered profit maximization but he did not expound on this example in his study.

PROBLEM SETTING

The manager of a terminal elevator is continuously faced with decisions related to buying and selling grain. His offers to buy grain are based on the relative quality attributes of each

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[3] T. E. Baynham, Jr., "Linear Programming--For Filling Orders at Grain Storage Elevators," The Northwestern Miller, CCLXV, No. 23 (October 30, 1961), p. 30.

[4] Wendell Clithero, "Computers for Wheat Blends, Purchasing," The Southwestern Miller, XLIII, No. 10 (May 1, 1964), p. 11-A.

[5] Joseph E. Unger, "Application of Linear Programming to Milling Problems Which Involve Blending of Wheat" (unpublished master's thesis, Kansas State University, 1957), pp. 38-44.

lot and the usefulness of each lot in blending to make grades with specified requirements. Sales are made based on these grades. Both the stocks already on hand as well as the stocks available must be considered in establishing a blending policy.

With the objective of maximizing profits, the following questions arise in establishing an optimum merchandising policy in a given situation:

(1) How many bushels of each lot should be used or purchased?

(2) How many bushels of each grade should be sold?

(3) How should the various lots be blended to make each of the grades sold?

(4) How sensitive is the policy to changes?

These questions will be explored assuming that the limited number of bushels available does not exceed the elevator capacity.

NECESSARY DATA

Four major areas will be considered in the initial analysis of the grain merchandising model: (1) availability and cost of the grain in each of the several bins or lots, (2) quality characteristics of each lot, (3) grade requirements and restrictions, and (4) grade prices and demand limitations.

Cost and Availability

For purposes of this study, 8 lots of corn of representative quality are assumed to be available. The quantity of each lot of

corn and the initial price of each lot are shown in Table 1. The price represents the market value of each lot of corn in dollars per bushel of 56 pounds at the terminal elevator.

Table 1. Supply and prices of corn available.

Lot	Quantity (bu.)	Market price ($ per bu.)
1	38,000	1.39
2	32,000	1.40
3	12,000	1.36
4	15,000	1.28
5	29,000	1.39
6	9,000	1.35
7	9,000	1.37
8	12,000	1.23

Quality Characteristics

In this study, it is assumed that the quality characteristics of each lot of corn are sufficiently known that the various lots can be blended with predictable accuracy. Table 2 displays the quality characteristics of each lot or corn available. Only those characteristics that are important in determining grade or price need be considered. For corn these factors are moisture, total damage, foreign material, heat damage, and odor. Modern instrumentation and sampling techniques permit accurate, rapid, and economical measurement of these factors by personnel experienced in operating a grain elevator.

Table 2. Characteristics of the grade determinants of each lot of corn.

Lot	Moisture %	Total Damage %	Foreign Material %	Heat Damage %	Odor Present
1	14.9	2.9	3.5	.05	None
2	14.5	4.3	1.9	.05	None
3	14.4	5.3	6.0	.05	None
4	16.0	20.0	4.0	.05	Sour
5	15.9	3.1	4.0	.05	None
6	14.8	6.8	3.0	.90	Musty
7	13.4	7.6	2.8	.05	None
8	13.8	40.0	5.0	5.00	Sour

Grade Requirements

The grade designations and grade requirements used in this study are presented in Table 3. These grades and specifications were supplied by a major grain merchandiser as representative of their actual corn merchandising activities.

Table 3. Grade designations and grade requirements of blended corn.

Grade Designation	Moisture Max. %	Total Damage Max. %	Foreign Material Max. %	Heat Damage Max. %	Odor Allowable
1	15.5	5.0	2.8	0.20	None
2	15.5	5.0	4.0	0.20	None
3	15.5	7.3	4.0	0.50	None
4	15.5	10.5	5.0	1.00	None
5	15.5	16.5	7.0	3.00	None
6	15.5	27.0	7.0	2.00	
7	15.5	No max	7.0	3.00	

Prices and Demand Limitations

Demand in the market may limit the quantity of some of the grades that can be sold. Within these limitations the demand faced by the firm is perfectly elastic. The demand limitations assumed in this study and the price of each grade are shown in Table 4. For example, the price of grade 6 is $1.28 per bushel and no more than 25,000 bushels may be sold at this price. Any quantity equal to or less than 25,000 bushels may be sold without affecting price. The price of each grade is indicative of the market value of that grade in the given time frame.

Table 4. Prices and demand limitations faced by the firm.

Grade Designation	Market Value	Upper Damand Limitations
	($ per bu.)	(max. number of bu.)
1	1.40	---
2	1.385	100,000
3	1.37	---
4	1.35	---
5	1.33	150,000
6	1.28	25,000
7	1.23	10,000

FORMULATION OF THE MODEL

After the necessary data have been outlined, the next step is formulating the model. In that the mathematical technique of linear programming has been chosen in this study, the data

described above must be assembled into a set of linear equations and linear inequalities. The difficult task lies in insuring that these linear equations and linear inequalities adequately and accurately define the conditons or situation under study.

Identification of Variables

To facilitate the statement of the problem in mathematical terms, the following variables and coefficients are defined:

X_{ij} = the number of bushels of the i^{th} lot used to make the j^{th} grade

X_{i0} = the number of bushels of the i^{th} lot sold without mixing

X_i = the total number of bushels of the i^{th} lot used or purchased

Y_j = the number of bushels of the j^{th} grade sold

a_{ki} = the k^{th} quality characteristic of the i^{th} lot

b_{kj} = the k^{th} quality characteristic of the j^{th} grade

c_i = the cost per bushel of the i^{th} lot

p_j = the price per bushel of the j^{th} grade

S_i = the supply of the i^{th} lot

D_j = the demand limits of the j^{th} grade

d_j = the demand requirements for the j^{th} grade

where

$i = 1, 2, \ldots, 8$ lots

$j = 1, 2, \ldots, 7$ grades

k = 1 (moisture), 2 (total damage), 3 (foreign material, 4 (heat damage), and 5 (odor) quality characteristics

Objective Function

The function to be optimized is called the objective function and depending on the circumstances can be either a minimum or a maximum. It is an expression of the problem objective in dollars and cents. In this study the goal is to maximize profits. In mathematical terms the objective function is stated as follows:

$$\sum_{j=1}^{7} p_j Y_j - \sum_{i=1}^{8} c_i X_i = Z\text{max}.$$

This function reflects the selling price per bushel of each grade and the cost or value per bushel of each lot used or bought.

Blending to Make Grades

The basic part of the model is the blending of the various lots of corn to meet the prescribed grade requirements. The structure of this problem is similiar to that of the feed formulation problem of determining the least cost combination of ingredients that can be used to meet predetermined product formulation specifications.

Linear inequalities or constraints are expressed in mathematical terms to take into account the per unit (bu.) contribution of each lot of corn to the requirement or restriction for each specification. Using the variables identified above the k^{th} specification constraint for the j^{th} grade takes the general form

$$\sum_{i=1}^{8} a_{ki} X_{ij} \leq b_{kj} Y_j$$

For example, the total damage constraint for grade 3 is stated as follows:

$$2.9\ X_{13} + 4.3\ X_{23} + \ldots + 40.0\ X_{83} \leq 7.3\ Y_3.$$

This specifies that the total damage level in lot 1 (2.9 percent) times the number of bushels of lot 1 used to blend into grade 3 plus the total damage level in lot 2 (4.3 percent) times the number of bushels of lot 2 used to blend into grade 3 plus . . . plus the total damage level in lot 8 (40.0 percent) times the number of bushels of lot 8 used to blend into grade 3 must be equal to or less than the total damage level restriction of grade 3 (7.3 percent) times the number of bushels of grade 3 after it is mixed. The moisture, foreign material, and heat damage requirements are also expressed in terms of maximum percentages and the corresponding constraints are handled in a similar manner.

The odor restraint takes a different form. While the other requirements are in terms of composition control comparable to nutrient control in a feed formulation problem, the odor restraint is one of utilization control. It is assumed that when no more than 20 percent from any stocks designated sour or 40 percent from any stocks designated musty or proportional quantities of sour and musty stocks are utilized the resulting mix will have no odor. A restriction considering only the quantity of sour stocks could be expressed by the inequality

$$\sum \text{sour stocks} \leq .20Y \quad , \qquad (1)$$

and a restriction considering only the quantity of musty stocks could be expressed by

$$\sum \text{musty stocks} \leq .40Y \quad , \qquad (2)$$

If inequality (1) is multiplied through by 2 the resulting inequality is

$$2\sum \text{sour stocks} \leq .40Y \quad . \qquad (3)$$

Comparing inequalities (2) and (3) indicates that 2 times the allowable quantity of the sour stocks is equivalent to the allowable quantity of the musty stocks, since both are less than or equal to the same quantity, .40Y. The proportional restriction considering both musty and sour stocks then is

$$2\sum \text{sour stocks} + \sum \text{musty stocks} \leq .40Y \quad , \qquad (4)$$

and is the only restraint that is needed for the odor specification. If there were no musty stocks the restraint would reduce to (1) while if there were musty stocks but no sour stocks the restraint would reduce to (2). The constraint then is formed by assigning a coefficient of 2 to stocks designated sour, a coefficient of 1 to stocks designated musty, and zero coefficients to those stocks which have no odor. In this problem lot 4 and lot 8 are sour and lot 6 is musty so the constraint becomes

$$2X_{4j} + X_{6j} + 2X_{8j} \leq .40Y_j \quad .$$

This constraint applies only to grades 1 through 5. There is no odor restriction on grades 6 and 7.

Necessary also in defining the blending portion of the problem is a quantity control or material balance equation. This equation takes the form

$$\sum_{i=1}^{8} X_{ij} = Y_j$$

and simply states that the quantity of the j^{th} grade mixed is the sum of the quantities of each of the lots that are being blended into the j^{th} grade.

<u>Quantity Restrictions</u>

Constraints are necessary to define the quantities of stocks on hand or available for purchase. These supply availability constraints take the form

$$\sum_{j=1}^{7} X_{ij} + X_{i0} \leq S_i \quad ,$$

indicating that the sum of the quantities of the i^{th} lot blended into each of the 7 grades plus the quantity of the i^{th} lot that is sold without mixing cannot exceed the quantity of the i^{th} lot on hand or available for purchase. This is an obvious statement but one that is necessary in terms of the model formulation. The availability constraint for lot 6, for example, is

$$X_{61} + X_{62} + \ . \ . \ . + X_{67} + X_{60} \leq 9{,}000 \text{ bu.}$$

For the purposes of this study it is assumed that demand in the market limits the quantities of certain of the grades that can be sold. These limitations must be reflected in the model and take the form

$$Y_j \leq D_j \ .$$

The constraint expressing the demand limitation of 10,000 bushels of grade 7 is

$$Y_7 \leq 10{,}000 \text{ bu.}$$

Limitations are not present for all grades but where limitations do exist they are expressed in a like manner.

It may be necessary or desirable for reasons not explored here to meet a minimum demand level for a certain grade. Such a requirement would enter the model in the form of a constraint,

$$Y_j \geq d_j \ ,$$

indicating that a minimum quantity of the j^{th} grade must be blended.

Although not expressly considered in this study, it may be desirable under some circumstances to require that a specific lot or portion of a lot be sold or used in blending. For example, if it were deemed necessary to insure that all of the sour corn in lot 4 were used, the following constraint would be added to the model:

$$\sum_{j=1}^{7} X_{4j} + X_{40} = 15{,}000 \text{ bu.}$$

Similar constraints could be formed for any of the lots or combinations of the lots.

Transfer Equations

To facilitate the statement of the objective function and for ease in the use of the model and analysis in later stages transfer equations or material balance equations are added. They take the form

$$\sum_{j=1}^{7} X_{ij} + X_{i0} = X_i$$

and serve to express in terms of a single variable (X_i) the total quantity of the i^{th} lot used.

The inequalities and equations necessary to meet the requirements of blending each grade and the basic structure of the model are shown in matrix format in Appendix I.

The non-negativity restrictions, those which require that the variables be non-negative, are taken care of by the linear programming algorithm and do not have to be explicitly expressed.

SOLUTION OF THE MODEL

The linear programming grain merchandising model begins with a set of constraints that describe both the limitations on resources and certain requirements or specifications that must be met. Since there are many more variables or unknowns than there

are equations and restrictions, an infinite number of solutions is possible. The problem is to select from this infinite number a solution that is optimum.

The objective function which describes the total profit is the driving force that is used to define the optimum solution. The goal is to maximize the value of the objective function (total profit) subject to the constraints. The optimum solution was determined on a high-speed electronic computer (IBM 360/50) using the standard linear programming code designed for that computer. Other computers of suitable size could have been used provided an appropriate program were available.

Technically, the problem is solved by iteratively computing the solution to the set of simultaneous linear equations and inequalities defining the model. The profit is increased by examining successive combinations of alternatives until the solution is the most profitable one. The theory of linear programming will not be discussed here except to state that the optimum solution for the model is determined.[6] Subject to the restrictions and demands of the model, <u>no</u> other quantities of the various grades sold and <u>no</u> other blending arrangement will increase the profit of the operation.

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[6] For a more complete explanation of linear programming two excellent references are G. Hadley, <u>Linear Programming</u> (Reading, Mass.: Addison-Wesley Pub. Co., 1962) and Kurz Meisels, <u>A Primer of Linear Programming</u> (New York: New York University Press, 1962).

Interpretation of the Answers

The basic or fundamental solution to a linear programming problem states the level of each activity that will yield the maximum profit while meeting all the restrictions and specifications and it gives the value of the maximum profit. In the grain merchandising model this solution states the quantity of each grade that should be sold and the quantity of each lot that should be used to blend each grade. This solution, given the initial prices and restrictions, is presented in Table 5. The optimum policy is to sell 79,761 bushels of grade 1; 45,928 bushels of grade 2; 13,800 bushels of grade 4; and 9,453 bushels of grade 6. It is not profitable at the existing prices to blend in all of lot 7; 7,058 bushels of lot 7 are sold without mixing. The make-up of each grade is also indicated in the table. For example, the 9,453 bushels of grade 6 sold are blended by mixing 565 bushels from lot 3; 5,164 bushels from lot 4, and 3,724 bushels from lot 8. The make-up of the other grades sold is similarly presented. The maximum profit from this blending policy is $1,996.86.

The one solution giving the single best policy for the problem as stated is often only the beginning of obtaining information about the situation under study. With very little extra effort and little more computer time considerable additional information can be derived from a linear programming solution. This additional information is at least useful secondary information and in some cases may be as important as the statement of

Table 5. Optimum solution with prices and restrictions as initially formulated

Lot	Price	Grade Sold 1 1.40	2 1.385	3 1.37	4 1.35	5 1.33	6 1.28	7 1.23	Sold As Is	Total Quantity
		(bu.)	(bu.)	(bu.)	(bu.)	(bu.)	(bu.)	(bu.)	(bu.)	(bu.)
1	1.39	31,305	6,695							38,000
2	1.40	32,000								32,000
3	1.36		3,363		8,072		565			12,000
4	1.28	4,239	4,055		1,542		5,164			15,000
5	1.39		29,000							29,000
6	1.35	9,000								9,000
7	1.37	3,217	2,815		2,968					9,000
8	1.23				1,218		3,724		7,058	12,000
Total		79,761	45,928		13,800		9,453		7,058	156,000

Z = $1,996.86 maximum profit from blending

optimum policy because it defines boundaries within which profitable trading can occur.

This is particularly true in the grain trade. Buying and selling grain are continuous simultaneous operations with changes in each occurring constantly. Naturally, many uncertainties exist. Basic to the structure of the model presented here is a degree of uncertainty associated with the grade quality characteristics of grain. Because of technical conditions in grain handling, grain never becomes a homogeneous mixture regardless of how many times it is mixed and handled. These uncertainties point out the importance of using the model as a guide. Supplementary information from linear programming solutions can be useful management data if interpreted with care.

Price Sensitivity

Once the optimum policy is known, it is valuable to have information such as the sensitivity of this policy to price fluctuations. There is a range over which objective function variations do not cause a change in the basic solution. These ranges of validity are not obvious from the objective function or the optimum solution, but they can be easily determined from the model.

Sensitivity of the optimum policy to changes in the price of each grade is indicated in Table 6. These price ranges measure the price change that can occur for each grade, taken one at a time, without necessitating a change in the optimum policy. For example, the price range for grade 1 is \$1.3985 to \$1.4074. In

more practical terms, considering the fact that the price of cash corn is quoted in terms of $\frac{1}{4}$ cent per bushel the range is \$1.40 to \$1.40$\frac{1}{2}$. This indicates that the price of grade 1, which is presently \$1.40 per bushel, can fluctuate within the price range of \$1.40 to \$1.40$\frac{1}{2}$ without changing the optimum policy, if all other prices remain stable. When the price drops below the lower limit, less of the grade should be sold and when the price rises above the upper limit more of the grade should be sold. Similar price ranges are given for grades 2, 4, and 6.

The grades which are not in the optimum policy, grades 3, 5, and 7, clearly have only an upper price limit, they already are too low in price. Regardless of how much the price drops below the present level, no change in the optimum solution will result. The upper limit in this case is the price at which the grade should become a part of the selling policy. The price of grade 5, for example, must be greater than \$1.3363 before grade 5 should be offered for sale.

Table 6. Current price of each grade and the price range over which the current policy would not be changed.

Grade	Current Price	Price Range: Lower Limit	Price Range: Upper Limit
	(\$ per bu.)	(\$ per bu.)	(\$ per bu.)
1	1.40	1.3985	1.4074
2	1.385	1.3819	1.3867
3	1.37	***	1.3732
4	1.35	1.3489	1.3503
5	1.33	***	1.3363
6	1.28	1.2798	1.2860
7	1.23	***	1.2362

In an actual situation, of course, prices generally will not change for only one grade at a time. In merchandising a single commodity, one would expect that the prices of all grades will generally move together, however, there will be price fluctuations with respect to each grade. A relative price change of a quarter of a cent or greater may necessitate a change in the optimum policy. Price ranges can prove valuable in determining the sensitivity of a policy to these price fluctuations. When price changes are widespread, it may be necessary to re-run the model to establish a new optimum policy. The optimum policy under one set of circumstances may not be optimum under another set of circumstances.

So far, only changes in selling prices have been considered. However, the same principles apply to changes in the cost of various lots used for blending.

Sensitivity of the optimum policy to changes in the cost or price of each lot is shown in Table 7. Lots 1 through 7 are used to the extent available, therefore, they do not have a bounded lower price range. Regardless of how much lower the price may be, the usage of these lots cannot be increased. The upper limit is the highest price at which all of the lot will continue to be used. For example, all of the 15,000 bushels of lot 4 available should continue to be used as long as the price of lot 4 is not greater than \$1.3069; all other prices remaining unchanged. All of lot 8 is not used in the blending policy, therefore, it has both a lower and an upper price limit. Below \$1.2198 more of lot 8 should be used, while above \$1.2303 less of lot 8 should be used.

Table 7. Current price of each lot and the price range over which the current policy would not be changed.

Lot	Current Price	Price Range Lower Limit	Price Range Upper Limit
	($ per bu.)	($ per bu.)	($ per bu.)
1	1.39	***	1.4018
2	1.40	***	1.4152
3	1.36	***	1.3628
4	1.28	***	1.3069
5	1.39	***	1.3944
6	1.35	***	1.3881
7	1.37	***	1.3865
8	1.23	1.2198	1.2303

Price ranges for each lot can be very helpful in establishing a purchasing policy. The model presented here not only considers what each lot is worth as it is, but it also determines the value of each lot in relation to the other lots on hand and available. High moisture grain normally will prove to be worth more to the merchandiser who has considerable dry grain on hand than it will to the merchandiser who already has an elevator full of wet grain. This will be reflected in the price ranges for each merchandiser. Price ranges are useful guides in evaluating the desirability of purchasing alternative lots.

The price range information indicates when a policy needs to be changed, but does not identify the complete nature or extent of the change required. At a price above the upper limit more of a grade should be sold, but the quantity to be sold and the blending of this grade are not evident. To complete the analysis of the change required the model must either be re-run with the new prices or a parametric study made over a range of prices.

Marginal Values

Having looked at the effects of a change in prices it can also be important to know what effects a change in the supply of a lot available will have on the optimum merchandising policy. As in the objective function and price ranges, there is also a range through which the quantity of a lot used can vary without changing the basic solution. However, the value of some of the variables will change, and there is a change in the total profit. Of interest then is a measurement of the change in total profit and the limits of the change in the quantity used or bought for which this measurement is valid.

The concept of marginal value can appropriately be applied in studying the effect on profit of a change in the usage or availability of a given lot. The marginal value of a lot is defined as the change in the value of the objective function (total profit) resulting from a unit change (a change of one bushel) in the quantity of the lot used. The marginal value of each lot in the situation under study and the quantity range over which they are applicable are shown in Table 8. For example, the marginal value of lot 2 is $.01521 per bushel and the applicable range is from 20,998 bushels to 39,406 bushels. For every bushel less than the original 32,000 bushels used, the profit will be reduced by just over a cent and a half a bushel until the level of 20,998 bushels is reached. For every bushel available over 32,000 bushels, profit can be increased by a cent and a half up to 39,406 bushels. Thus an increase of 1,406 bushels in the supply of lot 2 available could increase profit by $21.38. The marginal values of the other

lots are similarly interpreted.

Table 8. Marginal value of each lot and the range over which the marginal value is applicable.

Lot	Marginal Value ($ per bu.)	Original Solution Level (bu.)	Range Lower Limit (bu.)	Range Upper Limit (bu.)
1	.01184	38,000	30,812	48,677
2	.01521	32,000	20,998	39,406
3	.00288	12,000	4,099	21,142
4	.02699	15,000	9,979	23,258
5	.00444	29,000	10,740	55,455
6	.03812	9,000	0	14,949
7	.10656	9,000	5,566	38,822
8	.00034	4,942	4,006	4,942

The ranges for the marginal values shown above hold only when the quantity of one lot is changed at a time. It is possible, however, to reformulate the model to change prices of 2 or more lots at the same time to determine what the joint impact of such changes will be. The marginal values thus determined provide important management guides. Suppose that the quantities available were only estimates with uncertainties involved. The range of a marginal value gives an indication of how much the estimate can vary with a given effect on profit without requiring a change in the basic policy.

A related question is the effect on profit of an increase in supplies available. If an additional 10,000 bushels of each lot were made available and the merchandiser only has capacity to handle a total of 8,000 more bushels, the marginal values are a useful guide in determining which lot to increase. It is

evident from the data used in this study that lot 7 with a marginal value of $.10656, more than 10 cents per bushel, contributes most to total profit. Obviously lot 7 should be selected. Since the marginal value is valid up to 38,822 bushels, other factors unchanged, 8,000 bushels can be used.

ALLOCATION AT THE FIRM LEVEL

In a free enterprise marketing situation it must be recognized that many factors are subject to continuous and sometimes substantial change. A discussion of the fundamental solution and price range information is based on the assumptions that one thing is changed at a time and that the changes are not great enough to cause a change in the basic solution. By relaxing these assumptions it is possible to better understand the structure of the situation under study and how it might be affected by changes. Parametric programming, which is a technique for investigating the effect on the optimum linear programming solution of a sequence of proportionate changes in one or more of the elements of a single row or column of the matrix[7], allows the freedom to study changes. It is one of the best examples of the expanding usefulness of linear programming to management.

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[7] International Business Machines. An Introduction to Linear Programming . . . Data Processing Application Manual. (White Plains, N. Y.; I.B.M., 1964), p. 20.

Framework of Analysis

Most agricultural marketing situations are considered to represent near perfect competition. In such a case the decisions made by a buyer or a seller have no measurable effect upon market prices. Knowing this it is of interest to know what the firm's decision should be to maximize profits at various prices of a particular grade or lot.

A grain merchandiser is both a buyer and a seller. Supply curves can be developed to assist the grain merchandiser by observing the effect of a change in the price of a grade on the quantity of that grade which should be offered for sale. The supply curve shows the optimum quantities of a grade which should be offered for sale at alternative prices. Likewise, demand curves can be developed by observing the effect of a change in price of a certain lot on the quantity of that lot which the grain merchandiser should purchase. The demand curve shows the optimum quantities of a lot which should be purchased at alternative prices.

Within the framework of the model presented here both supply curves and demand curves can be developed by using parametric linear programming. The functions obtained by this technique are normative in the sense that they indicate what should be done in keeping with the goal of maximizing profits. They are not predictive in the sense that they will indicate what actually will

be supplied or purchased.[8] However, they can serve as useful management guides.

Supply Functions

In a variable-price or parametric linear programming analysis of the price of a particular grade in a given situation, prices for which the merchandising policy should change are computed. In this study the price was changed by increments of a quarter of a cent. Thus the scale of price-quantity relationships revealed by the analysis constitutes a normative supply function.

The supply function derived by parametric linear programming can be formalized as follows, using the variables previously defined:

$$Y_A = f(p_j, c_i, S_i, a_{ki}, b_{ki})$$

where again

$i = 1,2, \ldots, 8$ lots

$j = 1,2, \ldots, 7$ grades

$k = 1,2, \ldots, 5$ quality characteristics.

The quantity of grade A to be sold is not considered as simply a function of the price of grade A. The supply function also considers the prices of the other grades; the prices, the supplies,

[8]Ronald D. Krenz, Ross V. Baumann, and Earl O. Heady. "Normative Supply Functions by Linear Programming Procedures," Agricultural Economics Research, XIV (January, 1962), pp. 14-15.

and the quality characteristics of the various lots of grain available; and the quality restrictions or requirements of all the alternative grades. This fact is taken into account by considering that all factors are held constant while varying only the price in question.

In this study two of the supply functions were developed. The price-quantity relationships for grade 1, which was a part of the initial optimum policy, are summarized in Table 9, and the price-quantity relationships for grade 3, which was not a part of the initial optimum policy, are summarized in Table 10. These supply functions are presented graphically in Figures 1 and 2.

Table 9. Price-quantity relationships for grade 1 with all other factors constant.

Price Range	Quantity
($ per bu.)	(bu.)
$1.38\frac{1}{2}$	----
$1.38\frac{3}{4}$	4,393
1.39	24,042
$1.39\frac{1}{4}$ - $1.39\frac{1}{2}$	68,333
$1.39\frac{3}{4}$	79,548
1.40 - $1.40\frac{1}{2}$	79,761
$1.40\frac{3}{4}$	82,817
1.41 - $1.42\frac{1}{2}$	84,112
$1.42\frac{3}{4}$ - 1.43	86,121

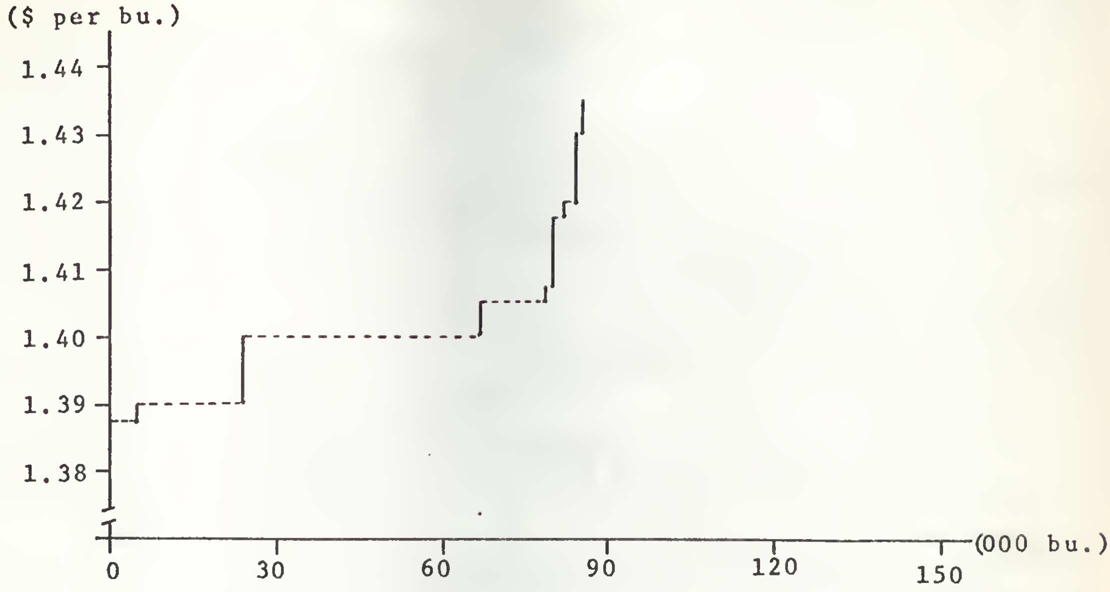

Fig. 1. Supply function for grade 1.

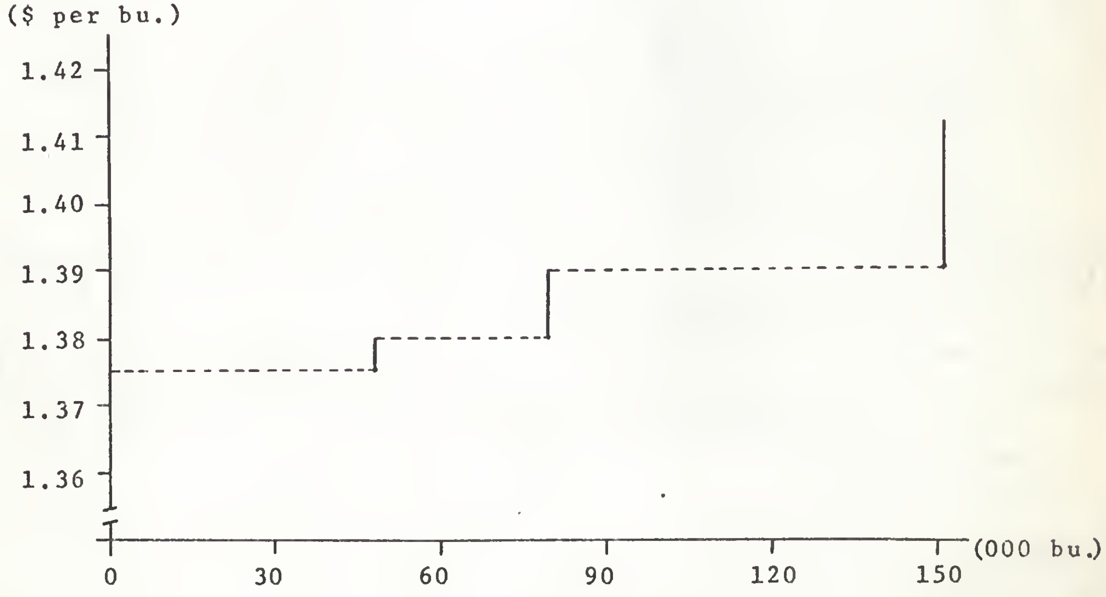

Fig. 2. Supply function for grade 3.

Table 10. Price-quantity relationships for grade 3 with all other factors constant.

Price Range	Quantity
($ per bu.)	(bu.)
$1.37\frac{1}{4}$	---
$1.37\frac{1}{2}$	47,382
$1.37\frac{3}{4}$ - $1.38\frac{3}{4}$	79,427
1.39 - 1.41	150,737

The supply function for grade 1, developed by varying the price of grade 1 while holding the other prices and factors fixed, reveals that none of grade 1 should be sold when the price is $\$1.38\frac{1}{2}$ or less. This is not difficult to see for the price of grade 2 is currently $1.38\frac{1}{2}$ and the quality restrictions are not as rigid. In the initial policy 79,761 bushels of grade 1 should be sold with the price at $1.40 per bushel. The price range information indicated that this quantity was the best policy as long as the price of grade 1 remained in the range $1.40 to $\$1.40\frac{1}{2}$, other prices and factors fixed, but did not indicate what should happen should the price fluctuate outside this range. The supply function indicates the optimum quantities at prices outside this range, and can in effect be viewed as a continuous extension of the price range information. The supply function for grade 3 is similarly presented.

Demand Functions

Although not as frequently discussed as supply functions, demand relationships at the firm level can also be revealed by parametric linear programming. In the analysis of a particular lot in a given situation, again prices for which the merchandising policy should change are computed. The demand function takes the same general form as the supply function, but here the quantity of a particular lot that should be bought or used at various prices of that lot is of interest:

$$X_B = g(p_j, c_i, S_i, a_{ki}, b_{ki}) \; .$$

Two of the demand functions were developed in this study; the demand function for lot 4, which was used in its entirety in the initial optimum policy and the demand function for lot 8, which was not used to the extent available in the initial optimum policy. The respective price-quantity relationships are summarized in Tables 11 and 12, and the demand functions are presented graphically in Figures 3 and 4. As with the supply function, the demand function for a particular lot is developed by varying the price of that lot while holding the other prices and factors fixed.

Table 11. Price-quantity relationships for lot 4 with all other factors constant.

Price Range	Quantity
($ per bu.)	(bu.)
$1.30\frac{1}{2}$	15,000
$1.30\frac{3}{4}$ - $1.31\frac{3}{4}$	9,979
1.32 - $1.32\frac{1}{4}$	7,780
$1.32\frac{1}{2}$ - $1.33\frac{3}{4}$	2,653
1.34	1,789

Table 12. Price-quantity relationships for lot 8 with all other factors constant.

Price Range	Quantity
($ per bu.)	(bu.)
$1.21\frac{1}{2}$	12,000
$1.21\frac{3}{4}$	8,399
1.22 - 1.23	4,942
$1.23\frac{1}{4}$ - $1.26\frac{3}{4}$	4,007
1.27	1,586

The demand function for lot 4 reveals that in the situation under study all 15,000 bushels of lot 4 available will be used up to a price of $\$1.30\frac{1}{2}$. This fact was pointed out in discussing the price range information. At a price of $\$1.32\frac{1}{2}$ only 2,653 bushels of lot 4 should be sought or demanded. The demand

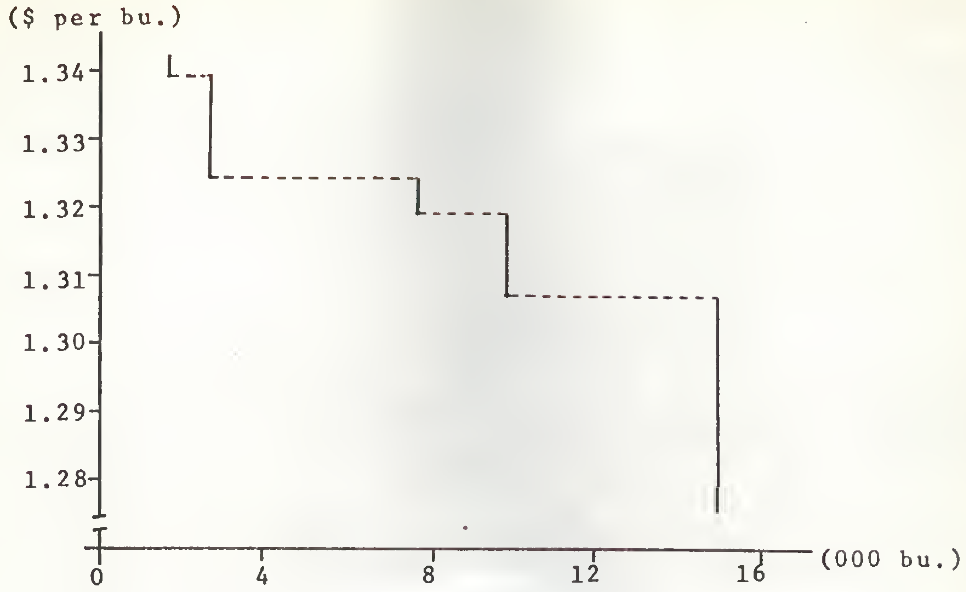

Fig. 3. Demand function for lot 4.

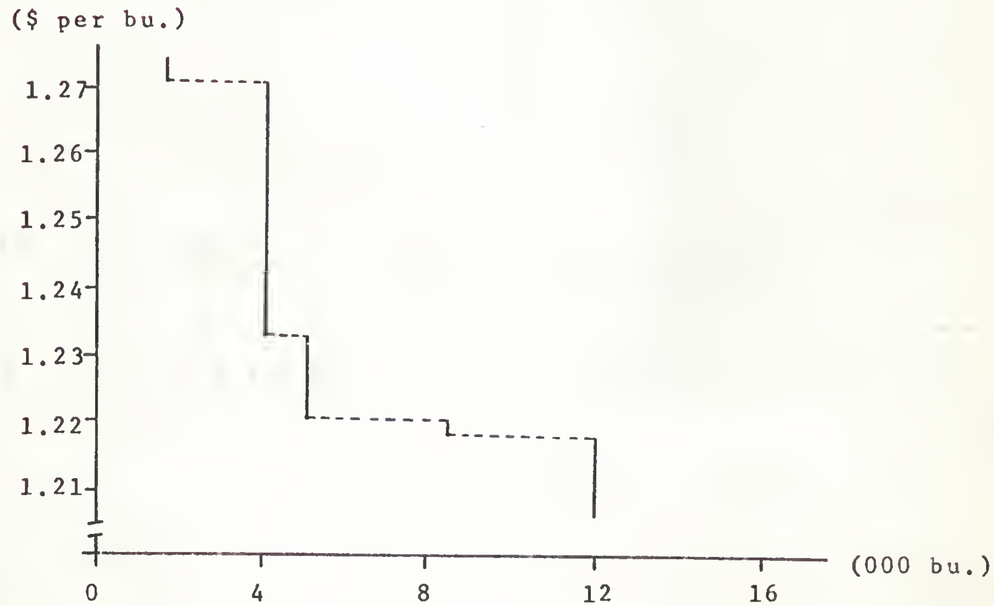

Fig. 4. Demand function for lot 8.

function for lot 8 is similarly presented. At the initial price of \$1.23 only 4,942 bushels of this lot are demanded. Not until the price falls to \$1.21$\frac{1}{2}$ or lower should all 12,000 bushels of lot 8 be used.

The demand relationships derived by parametric analysis may well be of more importance than the supply functions. In the model presented in this study, the entire quantity of a grade is sold with respect to a given basing point without regard to actual destination, and except for small fluctuations the prices of all grades will generally move together. Therefore, only small price changes around the original price of a grade will be very meaningful. The different lots, on the other hand, may be considered as coming from separate origins. Thus part of their cost will be transportation charges. As transportation rates are changed or alternative means of transportation are considered, it is not unreal to want to study a greater range for the cost of a particular lot. By using a parametric programming analysis the effects of a change in one or several costs simultaneously may be examined.

The resulting curves of both the supply and the demand functions are of a "stair-step" nature. The stair-step characteristic results from a finite number of alternatives, and rigid resource restrictions used in the programming calculations. The number of "steps and corners" is a function of the number of alternatives and restricting resources.[9]

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[9] Ibid., p. 17.

SUMMARY

The principle aim of a grain merchandiser is profit maximization. Thinking strictly "least-cost" does not insure maximum profit. This study had two main objectives: (1) to formulate a grain merchandising model which considers overall buying and selling policy or profit maximization, and (2) to develop some of the management information that can be derived from the basic model.

The mathematical technique of linear programming was chosen as the basis for a management model. The grain merchandising situation presented was formulated in terms of a linear programming problem and a basic solution was obtained. This solution indicated which grades should be sold, which lots should be used or purchased, how the various lots should be blended to make the grades, and the maximum profit for the given situation.

An important aspect of the model presented is the management information which can be derived from it. A model cannot eliminate all of the judgement associated with managerial decision-making. Nowhere is this more true than in grain merchandising where continuous changes and uncertainties are ever present. This points out the importance of using the model as a management <u>guide</u>. To enable the merchandiser to better understand the situation, sensitivity to changes was explored by looking at ranges on the objective function coefficients and at marginal values. Supply and demand functions for the firm were also derived using parametric analysis.

While only the fundamental aspects of grain merchandising were considered in developing this model, the basic structure offers the possibilities of several interesting extensions. The practical problems of grain drying or cleaning might be explored. The model also offers a framework in which the effects of transportation rate changes can be reviewed. Prospects for use of this "maximum-profit" model appear promising.

APPENDIX I

The equations and variables comprising the model were assembled in a systematic fashion by listing them all together. The variables were all transferred to the left-hand-side of the equation or inequality, leaving only a constant term on the right-hand-side. To conserve space and effort the variable name is "detached" and placed at the head of the column. The resulting array of coefficients is called a matrix. Figure 5 presents the basic matrix structure of the model while Tables 13 through 19 show the actual coefficients of the blending sub-matrices. In Figure 5 a plus sign represents a +1 coefficient and a minus sign represents a -1 coefficient.

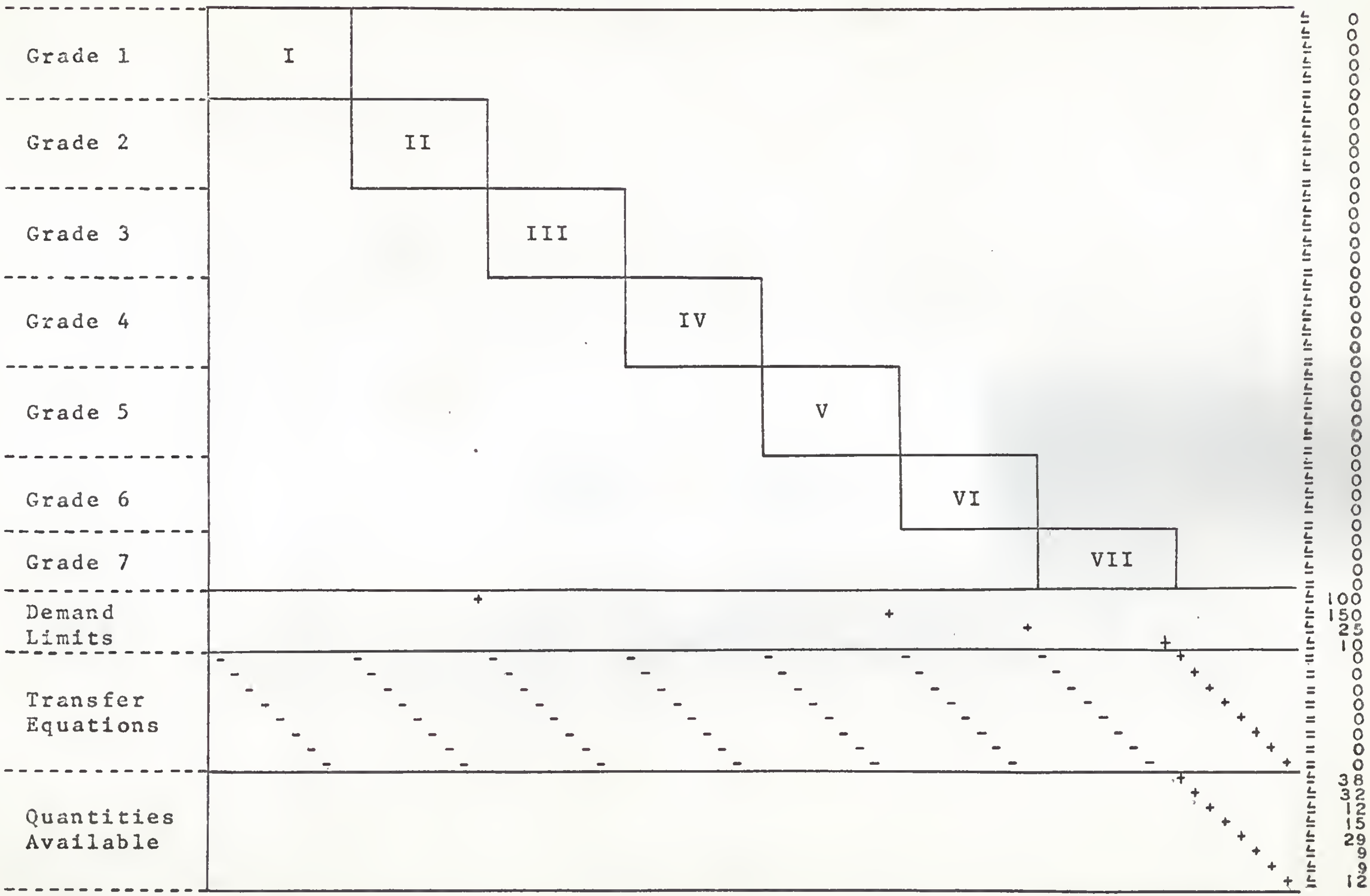

Fig. 5. Basic matrix structure of the grain merchandising model.

Table 13. Sub-matrix I, blending grade 1.

	X_{11}	X_{21}	X_{31}	X_{41}	X_{51}	X_{61}	X_{71}	X_{81}	Y_1	
Moisture	14.9	14.5	14.4	16.0	15.9	14.8	13.4	13.8	-15.5	≤ 0
Total damage	2.9	4.3	5.3	20.0	3.1	6.8	7.6	40.0	- 5.0	≤ 0
Foreign mat.	3.5	1.9	6.0	4.0	4.0	3.0	2.8	5.0	- 2.8	≤ 0
Heat damage	0.05	0.05	0.05	0.05	0.05	0.90	0.05	5.00	- 0.20	≤ 0
Odor				2		1		2	- 0.40	≤ 0
Mat. bal.	1	1	1	1	1	1	1	1	- 1	$= 0$

Table 14. Sub-matrix II, blending grade 2.

	X_{12}	X_{22}	X_{32}	X_{42}	X_{52}	X_{62}	X_{72}	X_{82}	Y_2	
Moisture	14.9	14.5	14.4	16.0	15.9	14.8	13.4	13.8	-15.5	≤ 0
Total damage	2.9	4.3	5.3	20.0	3.1	6.8	7.6	40.0	- 5.0	≤ 0
Foreign mat.	3.5	1.9	6.0	4.0	4.0	3.0	2.8	5.0	- 4.0	≤ 0
Heat damage	0.05	0.05	0.05	0.05	0.05	0.90	0.05	5.00	- 0.20	≤ 0
Odor				2		1		2	- 0.40	≤ 0
Mat. bal.	1	1	1	1	1	1	1	1	- 1	$= 0$

Table 15. Sub-matrix III, blending grade 3.

	X_{13}	X_{23}	X_{33}	X_{43}	X_{53}	X_{63}	X_{73}	X_{83}	Y_3
Moisture	14.9	14.5	14.4	16.0	15.9	14.8	13.4	13.8	$-15.5 \leq 0$
Total damage	2.9	4.3	5.3	20.0	3.1	6.8	7.6	40.0	$- 7.3 \leq 0$
Foreign mat.	3.5	1.9	6.0	4.0	4.0	3.0	2.8	5.0	$- 4.0 \leq 0$
Heat damage	0.05	0.05	0.05	0.05	0.05	0.90	0.05	5.00	$- 0.50 \leq 0$
Odor				2		1		2	$- 0.40 \leq 0$
Mat. bal.	1	1	1	1	1	1	1	1	$- 1 = 0$

Table 16. Sub-matrix IV, blending grade 4.

	X_{14}	X_{24}	X_{34}	X_{44}	X_{54}	X_{64}	X_{74}	X_{84}	Y_4
Moisture	14.9	14.5	14.4	16.0	15.9	14.8	13.4	13.8	$-15.5 \leq 0$
Total damage	2.9	4.3	5.3	20.0	3.1	6.8	7.6	40.0	$-10.5 \leq 0$
Foreign mat.	3.5	1.9	6.0	4.0	4.0	3.0	2.8	5.0	$- 5.0 \leq 0$
Heat damage	0.05	0.05	0.05	0.05	0.05	0.90	0.05	0.05	$- 1.00 \leq 0$
Odor				2		1		2	$- 0.40 \leq 0$
Mat. bal.	1	1	1	1	1	1	1	1	$- 1 = 0$

Table 17. Sub-matrix V, blending grade 5.

	X_{15}	X_{25}	X_{35}	X_{45}	X_{55}	X_{65}	X_{75}	X_{85}	Y_5
Moisture	14.9	14.5	14.4	16.0	15.9	14.8	13.4	13.8	-15.5 ≤ 0
Total damage	2.9	4.3	5.3	20.0	3.1	6.8	7.6	40.0	-16.5 ≤ 0
Foreign mat.	3.5	1.9	6.0	4.0	4.0	3.0	2.8	5.0	- 7.0 ≤ 0
Heat damage	0.05	0.05	0.05	0.05	0.05	0.90	0.05	5.00	- 3.00 ≤ 0
Odor				2		1		2	- 0.40 ≤ 0
Mat. bal.	1	1	1	1	1	1	1	1	- 1 $= 0$

Table 18. Sub-matrix VI, blending grade 6.

	X_{16}	X_{26}	X_{36}	X_{46}	X_{56}	X_{66}	X_{76}	X_{86}	Y_6
Moisture	14.9	14.5	14.4	16.0	15.9	14.8	13.4	13.8	-15.5 ≤ 0
Total damage	2.9	4.3	5.3	20.0	3.1	6.8	7.6	40.0	-27.0 ≤ 0
Foreign mat.	3.5	1.9	6.0	4.0	4.0	3.0	2.8	5.0	- 7.0 ≤ 0
Heat damage	0.05	0.05	0.05	0.05	0.05	0.90	0.05	5.00	- 2.00 ≤ 0
Mat. bal.	1	1	1	1	1	1	1	1	- 1 $= 0$

Table 19. Sub-matrix VII, blending grade 7.

	X_{17}	X_{27}	X_{37}	X_{47}	X_{57}	X_{67}	X_{77}	X_{87}	Y_8
Moisture	14.9	14.5	14.4	16.0	15.9	14.8	13.4	13.8	-15.5 ≤ 0
Foreign mat.	3.5	1.9	6.0	4.0	4.0	3.0	2.8	5.0	- 2.8 ≤ 0
Heat damage	0.05	0.05	0.05	0.05	0.05	0.90	0.05	5.00	- 0.20 ≤ 0
Mat. bal.	1	1	1	1	1	1	1	1	- 1 $= 0$

APPENDIX II

The demand and supply relationships derived through variable-price or parametric linear programming show the relationships between the price and the quantity of the particular grade or lot in question. It is obvious, however, that at each "step" the optimum policy should change. As the quantity of the grade or lot in question changes other quantities will change, as will the way in which these quantities are blended. Tables 20 through 24 show the total effect on the optimum policy at each "step" as the price of lot 8 is varied. Similar information could be presented for each parametric study, as could the new price sensitivity ranges and marginal value ranges which accompany each change in the optimum solution.

Table 20. Optimum solution when price for lot 8 ranges from \$1.21 to \$1.21$\frac{1}{2}$ and all other prices and restrictions are as initially formulated.

Lot	Price	Grade Sold 1 1.40 (bu.)	2 1.385 (bu.)	3 1.37 (bu.)	4 1.35 (bu.)	5 1.33 (bu.)	6 1.28 (bu.)	7 1.23 (bu.)	Total Quantity (bu.)
1	1.39	32,466	5,534						38,000
2	1.40	32,000							32,000
3	1.36		1,496		9,399		1,105		12,000
4	1.28	3,561	1,000		345		10,094		15,000
5	1.39		19,945		9,055				29,000
6	1.35	9,000							9,000
7	1.37	8,059	941						9,000
8	**		904		3,817		7,279		12,000
Total		85,086	29,820		22,616		18,478		

Table 21. Optimum solution when price for lot 8 is $\$1.21\frac{3}{4}$ and all other prices and restrictions are as initially formulated.

Lot	Price	Grade Sold 1 1.40 (bu.)	2 1.385 (bu.)	3 1.37 (bu.)	4 1.35 (bu.)	5 1.33 (bu.)	6 1.28 (bu.)	7 1.23 (bu.)	Total Quantity (bu.)
1	1.39	32,254	5,746						38,000
2	1.40	32,000							32,000
3	1.36		2,533		8,674		793		12,000
4	1.28	3,685	3,130		934		7,251		15,000
5	1.39		21,260		7,740				29,000
6	1.35	9,000							9,000
7	1.37	7,174	1,826						9,000
8	**				3,170		5,229		8,399
Total		84,113	34,495		20,518		13,273		

Table 22. Optimum solution when price for lot 8 ranges from $1.22 to $1.23 and all other prices and restrictions are as initially formulated.

Lot	Price	Grade Sold							Total Quantity (bu.)
		1 1.40 (bu.)	2 1.385 (bu.)	3 1.37 (bu.)	4 1.35 (bu.)	5 1.33 (bu.)	6 1.28 (bu.)	7 1.23 (bu.)	
1	1.39	31,305	6,695						38,000
2	1.40	32,000							32,000
3	1.36		3,363		8,072		565		12,000
4	1.28	4,239	4,055		1,542		5,164		15,000
5	1.39		29,000						29,000
6	1.35	9,000							9,000
7	1.37	3,217	2,815		2,968				9,000
8	**				1,218		3,724		4,942
Total		79,761	45,928		13,800		9,453		

Table 23. Optimum solution when price for lot 8 ranges from $\$1.23\frac{1}{4}$ to $\$1.26\frac{3}{4}$ and all other prices and restrictions are as initially formulated.

Lot	Price	Grade Sold 1 1.40 (bu.)	2 1.385 (bu.)	3 1.37 (bu.)	4 1.35 (bu.)	5 1.33 (bu.)	6 1.28 (bu.)	7 1.23 (bu.)	Total Quantity (bu.)
1	1.39	31,259	6,741						38,000
2	1.40	32,000							32,000
3	1.36		3,367		8,633				12,000
4	1.28	4,266	4,064		1,649		5,021		15,000
5	1.39		29,000						29,000
6	1.35	9,000							9,000
7	1.37	3,023	2,802		3,175				9,000
8	**				1,303		2,704		4,007
Total		79,548	45,974		14,760		7,725		

Table 24. Optimum solution when price for lot 8 is $1.27 and all other prices and restrictions are as initially formulated.

		Grade Sold							
Lot	Price	1 1.40 (bu.)	2 1.385 (bu.)	3 1.37 (bu.)	4 1.35 (bu.)	5 1.33 (bu.)	6 1.28 (bu.)	7 1.23 (bu.)	Total Quantity (bu.)
1	1.39	31,017	6,983						38,000
2	1.40	32,000							32,000
3	1.36		1,746	2,476	7,778				12,000
4	1.28	4,407	2,484	5,858	1,486		765		15,000
5	1.39		12,170	16,830					29,000
6	1.35	9,000							9,000
7	1.37	2,013		4,127	2,860				9,000
8	**				1,174		412		1,586
Total		78,437	23,383	29,291	13,298		1,176		

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APPLICATION OF A LINEAR PROGRAMMING MODEL
TO GRAIN MERCHANDISING

by

EDWARD LOWELL JANZEN

B. S., Kansas State University, 1962

AN ABSTRACT OF A MASTER'S THESIS

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MASTER OF SCIENCE

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KANSAS STATE UNIVERSITY
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One of the most popular and useful techniques that management can turn to for assistance in the systematic approach to the formulation and solution of business problems is the mathematical technique of linear programming.

The purpose of this study was to formulate a linear programming model which considered profit maximization for a grain merchandising situation and to develop some of the management information that can be derived from the basic model.

The grain merchandiser is continuously faced with decisions related to buying and selling grain. His offers to buy grain are based on the relative quality attributes of each lot and the usefulness of each lot in blending to make grades with specified requirements. Sales are made based on these grades. Both the stocks already on hand as well as the stocks available must be considered in establishing a blending policy.

The mathematical technique of linear programming was chosen as the basis for a management model. The grain merchandising situation presented was formulated in terms of a linear programming problem and a basic solution was obtained.

This solution indicated which grades should be sold, which lots should be used or purchased, how the various lots should be blended to make the grades, and the maximum profit for the given situation.

An important aspect of the model presented is the use of the management information. A model cannot eliminate all of the judgment associated with managerial decision-making. Nowhere is

this more true than in grain merchandising where continuous changes and uncertainties are ever present. This points out the importance of using the model as a management guide. To enable the merchandiser to better understand the situation, sensitivity to changes was explored by looking at ranges on the objective function coefficients and at marginal values. Supply and demand functions for the firm were also derived using parametric analysis.